hence

$$\eta = \frac{T_h^5 - T_c^5 - (T_h + T_2 - T_c)^5 + T_2^5}{5(T_h - T_c)T_h^4} \qquad (11)$$

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Dynamical Equations for the Plane Change Maneuver of the Lunar Hopping Laboratory

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Nomenclature

A = piston area or foot contact point
c.m. = center of mass
d = propulsion cylinder displacement
g = gravitational constant
h = offset of c.m.$_P$ from leg axis measured in the x direction
K = isentropic constant
l = half length of HOLAB thrust leg
m, M = mass of thrust leg and pod, respectively
r = gear ratio with respect to leg and pod motion
z_P = leg displacement measured between foot and c.m.$_P$
γ = specific heat ratio of the working fluid
δ = hop plane angle
ζ = distance between c.m.$_L$ and c.m.$_v$
η = distance between c.m.$_L$ and c.m.$_P$
ϕ, θ, ψ = Euler angles used to specify vehicle orientation

Subscripts

c, c.m. = c.m. of subsystem and vehicle, respectively
d, e = disengagement and engagement, respectively
L, P, v = thrust leg, pod, and vehicle, respectively
x, y, z = parallel to x, y, and z axes, respectively

Superscripts

$f(t_0^+)$ limiting value of the dependent variable f at t_0 when t_0 is approached from a value greater than t_0
(t_0^-) = limiting value when t_0 is approached from a value less than t_0

Introduction

THE use of hopping motion to travel on the lunar surface was first introduced by Seifert.[1] The Lunar Hopping Laboratory (HOLAB) design is one of two basic transporter configurations studied recently.[2] It is intended to be a sophisticated mobility system with long-range and duration capabilities. Unlike simpler hopping transporters, HOLAB can change hopping planes without stopping vehicle motion between successive leaps. Plane changing is effected while the foot is in contact with the surface, i.e., time from the beginning of deceleration of one hop to the end of acceleration of the next. The necessary torques about the foot are created by rotating the vehicle center-of-mass (c.m.$_v$) out of the hop plane before disengagement occurs, and weight and inertia forces produce the desired plane change. The development of the equations of motion and transition relations for HOLAB between ballistic and foot-in-contact phases are of primary concern here.

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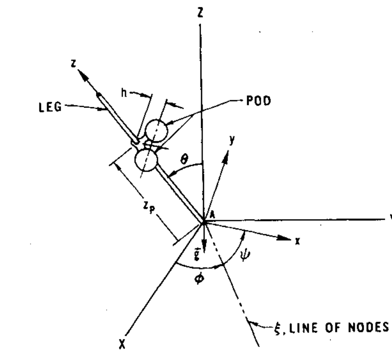

Fig. 1 Mathematical model for the analysis.

Equations of Motion for the Plane Change Maneuver

We wish to describe the motion of a rigid slender rod, sliding pod configuration about a fixed point under the influence of propulsion and gravity. The transition between ballistic flight and the foot-in-contact interval is also important but requires only the derivation and solution of algebraic equations (see the following section).

In the mathematical model (Fig. 1), each of the two cabins making up the pod of total mass M is represented by a sphere. These spheres are equal in mass and size and are of uniform density. The distance between their centers is 2.5 radii. The connecting mechanism containing a gear box is assumed to be massless; the mass of the foot also is ignored. Therefore, the pod accounts for all vehicle mass except that of thrust leg, which is a long, thin, homogeneous rod of mass m and length $2l$. The x,y,z axes are fixed to the vehicle at point A with the x axis normal to the z axis and in the plane of pod symmetry which also contains z. X,Y,Z represent an inertial reference frame. The only allowable pod rotations with respect to the body-fixed frame occur about the line of cabin centers. Such rotations are executed by the control system so that the crew will not be subjected to pitching motions and therefore will be able to take observational data during reorientation and acceleration maneuvers. This system has four degrees of freedom, three Euler angles and the linear position of the pod relative to point A. Since the propulsive force is a function of pod displacement only, the conservative form of Lagrange's equations can be applied.[3] The absence of crew pitch freedom can easily be treated by ignoring the pod moment of inertia about its y axis when deriving the differential equations of rotational motion. For this analysis, it is assumed that all masses and moments of inertia are known, and h is a constant parameter whose value is critical to efficient performance of this maneuver. In order to avoid a great deal of algebraic manipulation, the body-fixed axes were selected to coincide with principal leg axes, and these are parallel to principal pod axes. Noting that $v_{P_c}^2 = \dot{z}_P^2 + h^2(\omega_y^2 + \omega_z^2) + z_P^2(\omega_x^2 + \omega_y^2) - 2hz_P\omega_x\omega_z - 2h\dot{z}_P\omega_y$ permits the system kinetic energy to be written as

$$\begin{aligned} T = {} & I_x(\dot\phi S_\theta S_\psi + \dot\theta C_\psi)^2/2 + I_y(\dot\phi S_\theta C_\psi - \dot\theta S_\psi)^2/2 + \\ & I_z(\dot\phi C_\theta + \dot\psi)^2/2 + M\{\dot{z}_P^2 + z_P^2(\dot\phi^2 S_\theta^2 + \dot\theta^2) + \\ & h^2[\dot\psi^2 + \dot\theta^2 S_\psi^2 + \dot\phi^2(C_\psi^2 + C_\theta^2 S_\psi^2) + 2\dot\phi\dot\psi C_\theta - \\ & 2\dot\phi\dot\theta S_\theta S_\psi C_\psi] - 2hz_P[\dot\phi^2 S_\theta C_\theta S_\psi + \dot\theta\dot\psi C_\psi + \\ & \dot\phi\dot\psi S_\theta S_\psi + \dot\phi\dot\theta C_\theta C_\psi] + 2h\dot{z}_P[\dot\theta S_\psi - \dot\phi S_\theta C_\psi]\}/2 \end{aligned} \qquad (1)$$

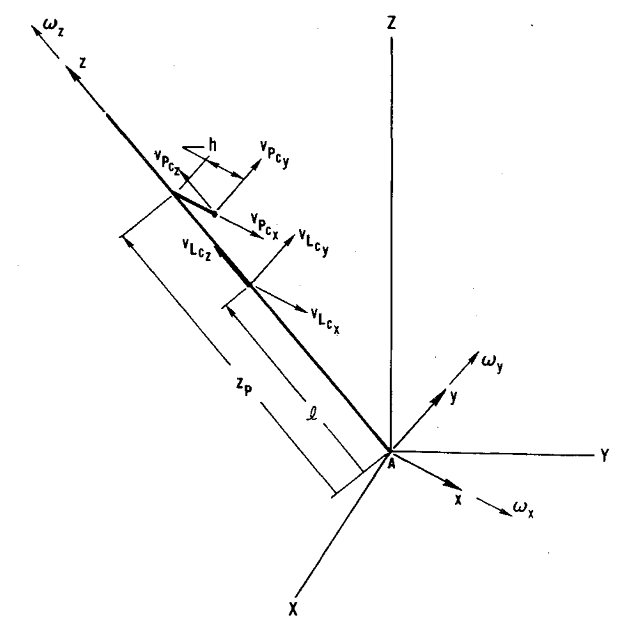

Fig. 2 Nomenclature for transition relations.

where $S_\theta = \sin\theta$, $S_\phi = \sin\phi$, $S_\psi = \sin\psi$, $C_\theta = \cos\theta$, $C_\phi = \cos\phi$, and $C_\psi = \cos\psi$; $I_x = I_{Lx} + I_{P_{cx}}$, $I_y = I_{Ly}$, $I_z = I_{Lz} + I_{P_{cz}}$; I_{Lx}, I_{Ly}, I_{Lz} = leg moments of inertia about point A, and $I_{P_{cx}}$, $I_{P_{cy}}$, $I_{P_{cz}}$ = pod moments of inertia about its c.m. The potential energy is immediately obtainable from Fig. 1,

$$V = mglC_\theta + Mg(z_P C_\theta + hS_\theta S_\psi) + V_p(z_P) \qquad (2)$$

where V_p is the propulsive potential energy. The thrust force will appear in the differential equations as $-\partial V_p/\partial z_P$, or explicitly as[2]

$$K(r/A)^{\gamma-1}/(z_P - z_{P_d} + d_d r)^\gamma$$

For the simulations performed here, the mass of working fluid in the cylinder is assumed constant throughout the entire plane-change maneuver. Therefore, K has a constant value in the time interval of interest. In the actual case, thermodynamic properties of this gas are changed at the end of deceleration. However, results indicate that plane-change performance is insensitive to small variations in propulsion parameters.

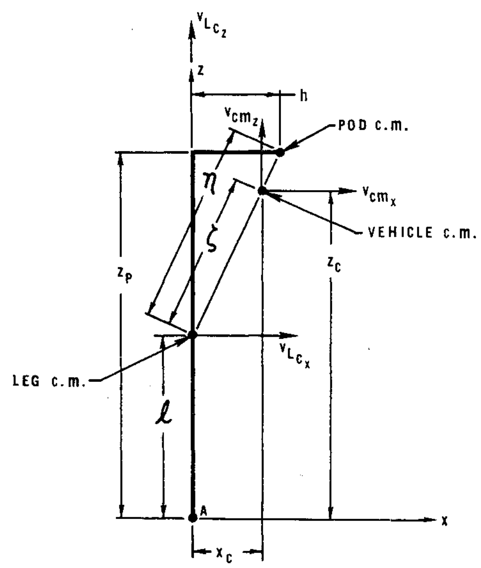

Fig. 3 Geometric relations between vehicle, pod, and leg centers of mass.

Table 1 Transition relations at disengagement

at t_d^-	at t_d^+
$v_{L_{cx}} = v_{P_{cx}} - (z_P - l)\omega_y$	$v_{L_{cx}} = l\omega_y$
$v_{L_{cy}} = v_{P_{cy}} - h\omega_z + (z_P - l)\omega_x$	$v_{L_{cy}} = -l\omega_x$
$v_{L_{cz}} = v_{P_{cz}} + h\omega_y$	$v_{P_{cx}} = z_P\omega_y$ $v_{P_{cy}} = h\omega_z - z_P\omega_x$

Taking the derivatives of expressions Eqs. (1) and (2) as required by the Lagrangian formulation gives the equations of motion. Since ϕ is cyclic, one of these equations is simply $\partial T/\partial\dot\phi$ = const, or

$$I_x(\dot\phi S_\theta^2 S_\psi^2 + \dot\theta S_\theta S_\psi C_\psi) + I_y(\dot\phi S_\theta^2 C_\psi^2 - \dot\theta S_\theta S_\psi C_\psi) + I_z(\dot\phi C_\theta^2 + \dot\psi C_\theta) + M[z_P^2\dot\phi S_\theta^2 - h\dot z_P S_\theta C_\psi + h^2(\dot\phi C_\psi^2 + \dot\phi C_\theta^2 S_\psi^2 + \dot\psi C_\theta - \dot\theta S_\theta S_\psi C_\psi) - hz_P(2\dot\phi S_\theta C_\theta S_\psi + \dot\psi S_\theta S_\psi + \dot\theta C_\theta C_\psi)] = \text{const}$$

Unfortunately, this type of simplification is not possible for the other three coordinates:

$$I_x[\ddot\phi S_\theta S_\psi C_\psi + \dot\phi\dot\psi S_\theta(C_\psi^2 - S_\psi^2) + \ddot\theta C_\psi^2 - 2\dot\theta\dot\psi S_\psi C_\psi - \dot\phi^2 S_\theta C_\theta S_\psi^2] - I_y[\ddot\phi S_\theta S_\psi C_\psi + \dot\phi\dot\psi S_\theta(C_\psi^2 - S_\psi^2) - \ddot\theta S_\psi^2 - 2\dot\theta\dot\psi S_\psi C_\psi + \dot\phi^2 S_\theta C_\theta C_\psi^2] + I_z(\dot\phi^2 S_\theta C_\theta + \dot\theta\dot\psi S_\theta) + M\{2z_P\dot z_P\dot\theta + z_P^2\ddot\theta - z_P^2\dot\phi^2 S_\theta C_\theta + h^2[\ddot\theta S_\psi^2 + 2\dot\theta\dot\psi S_\psi C_\psi - \ddot\phi S_\theta S_\psi C_\psi + 2\dot\phi\dot\psi S_\theta S_\psi^2 + \dot\phi^2 S_\theta C_\theta S_\psi^2] - hz_P[\ddot\psi C_\psi - \dot\psi^2 S_\psi + \ddot\phi C_\theta C_\psi - 2\dot\phi\dot\psi C_\theta S_\psi + \dot\phi^2 S_\psi(S_\theta^2 - C_\theta^2)] + h\ddot z_P S_\psi\} - mglS_\theta + Mg(-z_P S_\theta + hC_\theta S_\psi) = 0$$

$$I_z(\ddot\phi C_\theta - \dot\phi\dot\theta S_\theta + \ddot\psi) - (I_x - I_y)(\dot\phi^2 S_\theta^2 S_\psi C_\psi - \dot\phi\dot\theta S_\theta S_\psi^2 + \dot\phi\dot\theta S_\theta C_\psi^2 - \dot\theta^2 S_\psi C_\psi) + M\{h^2[\ddot\psi + \ddot\phi C_\theta - 2\dot\phi\dot\theta S_\theta S_\psi^2 - \dot\theta^2 S_\psi C_\psi + \dot\phi^2 S_\theta^2 S_\psi C_\psi] - 2h\dot z_P(\dot\theta C_\psi + \dot\phi S_\theta S_\psi) - hz_P(\ddot\theta C_\psi + \ddot\phi S_\theta S_\psi + 2\dot\phi\dot\theta C_\theta S_\psi - \dot\phi^2 S_\theta C_\theta C_\psi)\} + MghS_\theta C_\psi = 0$$

$$M[\ddot z_P - z_P(\dot\phi^2 S_\theta^2 + \dot\theta^2) + h(\ddot\theta S_\psi - \ddot\phi S_\theta C_\psi + \dot\phi^2 S_\theta C_\theta S_\psi + 2\dot\theta\dot\psi C_\psi + 2\dot\phi\dot\psi S_\theta S_\psi)] + MgC_\theta - K(r/A)^{\gamma-1}/(z_P - z_{P_d} + d_d r)^\gamma = 0$$

The coupling among coordinates and the nature of the propulsion law make these differential equations extremely non-linear in a rather complicated manner. In general, the motion is neither periodic nor of small amplitude, indicating the only practical method of solution is by application of computer techniques.

Transition Relationships

At the instant of disengagement t_d the foot contacts the lunar surface, and the pod is simultaneously unlocked and becomes free to move along the leg under the influence of propulsive, gravitational, and acceleration forces. Loss of all momentum and kinetic energy is experienced by the foot, but angular momentum about point A is conserved. Since the pod is released to move down the leg at time t_d^-, it does not experience any impact effects along the z-direction during this event. No displacement between pod and leg takes place in this instant, so that propulsive forces do no work on the pod. Therefore, the velocity along the leg of a point on the pod-leg connection mechanism which also lies on the z axis will not be affected by disengagement. Figure 2 illustrates the geometric relationships. Thus, conservation of angular momentum about point A takes the form

$$[(I_{Lc_x} + I_{Pc_x})\omega_x - mlv_{Lc_y} - Mz_Pv_{Pc_y}]\Big|_{t_d^-}^{t_d^+} = 0 \quad (3)$$

$$[(I_{Lc_y} + I_{Pc_y})\omega_y + mlv_{Lc_x} + M(z_Pv_{Pc_x} - hv_{Pc_z})]\Big|_{t_d^-}^{t_d^+} = 0 \quad (4)$$

$$[(I_{Lc_z} + I_{Pc_z})\omega_z + Mhv_{Pc_y}]\Big|_{t_d^-}^{t_d^+} = 0 \quad (5)$$

where only the values of the angular and linear velocity components are affected by disengagement. The pod-leg connecting point condition is expressed as

$$v_{Pc_z}(t_d^+) = v_{Lc_z}(t_d^-) - h\omega_y(t_d^+)$$

The necessary kinematic relations between leg and pod are listed in Table 1. If components of angular and linear velocity are known at t_d^-, then disengagement relations will uniquely determine all values of vehicle motion at time t_d^+.

At the end of the plane-change maneuver, engagement occurs over an infinitesimal time interval. Vehicle angular and linear momentum are conserved, and initial ballistic coordinate and velocity values at t_e^+ can be uniquely determined from conditions at t_e^-. Equating angular momentum components about point A at times t_e^- and t_e^+ gives conditions (3–5) with t_d everywhere replaced by t_e. Equating linear momentum components at times t_e^- and t_e^+ gives

$$[mv_{Lc_x} + Mv_{Pc_x}]\Big|_{t_e^-}^{t_e^+} = 0, \quad [mv_{Lc_y} + Mv_{Pc_y}]\Big|_{t_e^-}^{t_e^+} = 0$$

$$Mv_{Pc_z}\Big|_{t_e^-} = [mv_{Lc_z} + Mv_{Pc_z}]\Big|_{t_e^+}$$

The kinematic relations of Table 1 apply at engagement when t_d^- and t_d^+ are replaced by t_e^+ and t_e^-, respectively.

The primary objective of this analysis is to relate an initial ballistic flight plane to a new flight plane. These planes are defined by the c.m.$_v$ velocity components which lie in the XY-plane. Inertial velocity components $\dot{X}_{c.m.}(t_d^-)$, $\dot{Y}_{c.m.}(t_d^-)$, and $\dot{Z}_{c.m.}(t_d^-)$ are assumed to be given in addition to $z_P(t_d^-)$ and the Euler angles and their rates at time t_d^-. These c.m.$_v$ velocities must be converted to body-fixed coordinates before disengagement conditions can be calculated. This is done by using the well-known orthogonality transformation for Euler angles.[3] c.m.$_v$ velocity components are related to c.m.$_L$ components with the aid of Fig. 3,

$$v_{Lc_x} = v_{cm_x} - (z_c - l)\omega_y \quad (6)$$

$$v_{Lc_y} = v_{cm_y} - x_c\omega_z + (z_c - l)\omega_x \quad (7)$$

$$v_{Lc_z} = v_{cm_z} + x_c\omega_y \quad (8)$$

where

$$x_c = \zeta h/\eta, \quad z_c = [\zeta(z_P - l)/\eta] + l$$

$$\zeta = M\eta/(M + m), \quad \eta = [(z_P - l)^2 + h^2]^{1/2}$$

Values given by these expressions are used with disengagement conditions to convert c.m.$_v$ components into c.m.$_L$ and c.m.$_P$ components. At the subsequent engagement event, values of c.m.$_v$ velocity components must be obtained in terms of the inertial frame. First, these components are calculated in body-fixed coordinates from components of the c.m.$_L$ velocity at time t_e^+ by solving Eqs. (6–8). Then another orthogonality transformation is performed to obtain inertial coordinates. The expressions presented to this point completely describe the dynamical state of this vehicle model in the interval $t_d^- \le t \le t_e^+$. Given c.m.$_v$ velocity components and angular state with z_P at time t_d^-, these relationships can be solved to generate the entire state of vehicle dynamics at time t_e^+. Thus,

$$\delta(t_e^+) = \tan^{-1}[\dot{Y}_{c.m.}(t_e^+)/\dot{X}_{c.m.}(t_e^+)]$$

The desired value of plane change is just the difference of this and the initial plane angle $\delta(t_e^-)$.

Conclusions

The general dynamical motion of a lunar hopping vehicle is, at best, extremely complex. Only one simplified aspect of the dynamics problem has been considered here, development of the equations of motion for the plane-change maneuver. A general solution has not yet been discovered, but specialized numerical results are obtainable for given initial conditions. In order to design a navigational computer and control system for such a vehicle, the plane-change problem must be completely solved. This will require a considerable amount of effort beyond the present state of knowledge.

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A Guarded Disk-Type Sample Emissometer

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Nomenclature

P	= power applied to the sample heater
E	= voltage drop across the sample heater
I	= current
A_s	= total radiating area of the two sample disks
σ	= Stefan-Boltzman constant
T_s, T_0	= sample and cold cavity temperatures
ϵ_s	= total hemispherical emittance of the sample
ϵ_n	= total normal emittance
α_s	= absorptance of the sample for energy radiated by the cold cavity walls
q	= heat flow; q_l, heat leak through the lead wires; q_g, through the residual gas; q_r, net radiation loss through the edge of the sample and sample heater to the guard ring and cold cavity
w	= mass
c	= heat capacity
$\dot{T}$	= derivative of temperature with respect to time

Introduction

Knowledge of the total hemispherical emittance ϵ_s of materials at satellite temperature ranges is important for the thermal design of space or space simulation systems. Calorimetric methods for ϵ_s measurement are based on either the transient or the steady-state modes. When the transient mode is used, the accuracy of the measured ϵ_s depends upon accurate knowledge of the heat capacity, $c = c(T)$, of the specimen. Very often the specimen consists of a relatively thick coating on a thin metal substrate, in which case transient ϵ_s

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